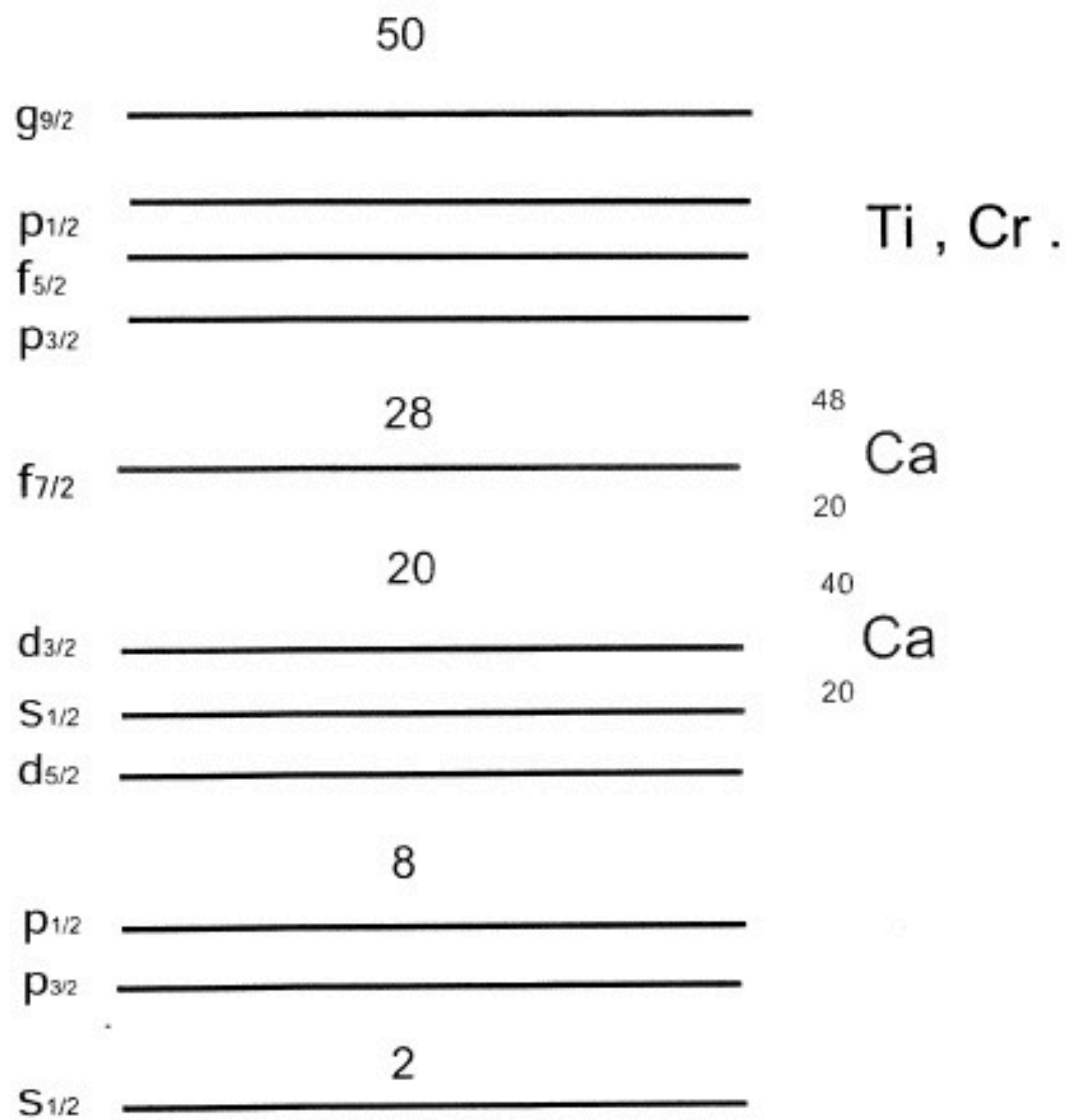
The fp shell



50
g9/2
p1/2
f5/2
p3/2
Ti , Cr .
28
48
Ca
20
f7/2
20
40
Ca
20
d3/2
s1/2
d5/2
8
p1/2
p3/2
2
s1/2

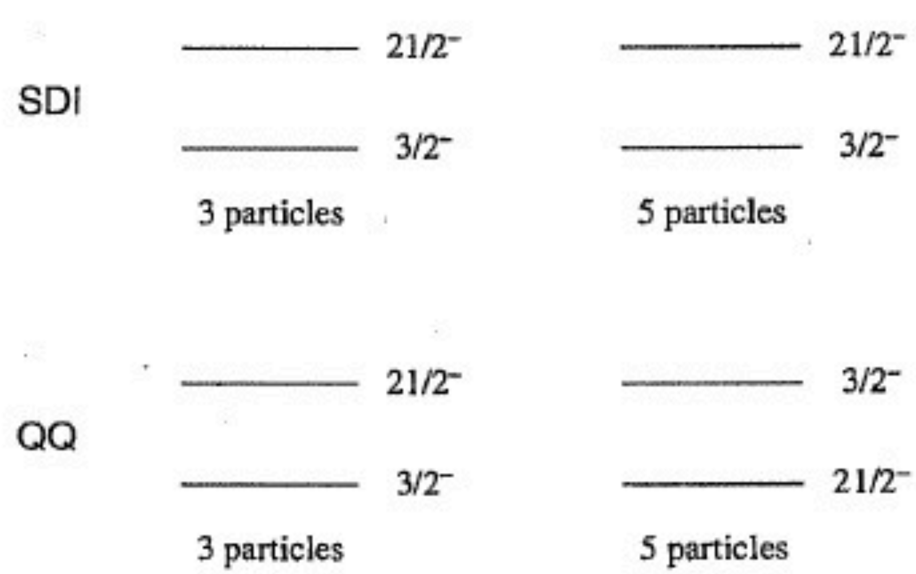
INTERESTING RESULTS FOR $g_{9/2}$ SHELL

(identical particles)

- 3 identical particles:

  We define: $\Delta E_3 = E_3(I_{\text{max}}) - E_3(I_{\text{min}}) = E_3(21/2) - E_3(3/2$

- 5 particles:
  - With seniority-conserving interaction: $\Delta E_5 = \Delta E_3$
  - With $Q \cdot Q$ interaction: $\Delta E_5 = -\Delta E_3$
  - In $f_{7/2}$, $\Delta E_5 = \Delta E_3$ even with $Q \cdot Q$



Topic 2. 4neutrons (holes) in g9/2-- 96Pd

I = 4

J_0	v_0	v=4	v=4	v=2
1.5	3	-0.045216	0.486667	0.284268
2.5	3	0.007841	0.392310	-0.181186
3.5	3	-0.619461	0.034481	0.176295
4.5	1	0.000000	0.000000	0.612373
4.5	3	-0.055655	-0.344807	0.344932
5.5	3	0.451454	0.259542	0.363442
6.5	3	0.574601	-0.298436	0.156447
7.5	3	0.240746	0.489004	-0.243006
8.5	3	-0.138306	0.305968	0.381690

- With seniority mixing, one $v = 4$ state remains pure:

I = 4

J_0	v_0	v=4
1.5	3	0.1222
2.5	3	0.0548
3.5	3	0.6170
4.5	1	0.0000
4.5	3	0.0000
5.5	3	-0.4043
6.5	3	-0.6148
7.5	3	-0.1597
8.5	3	0.1853

II. FURTHER RELATIONS

We can write

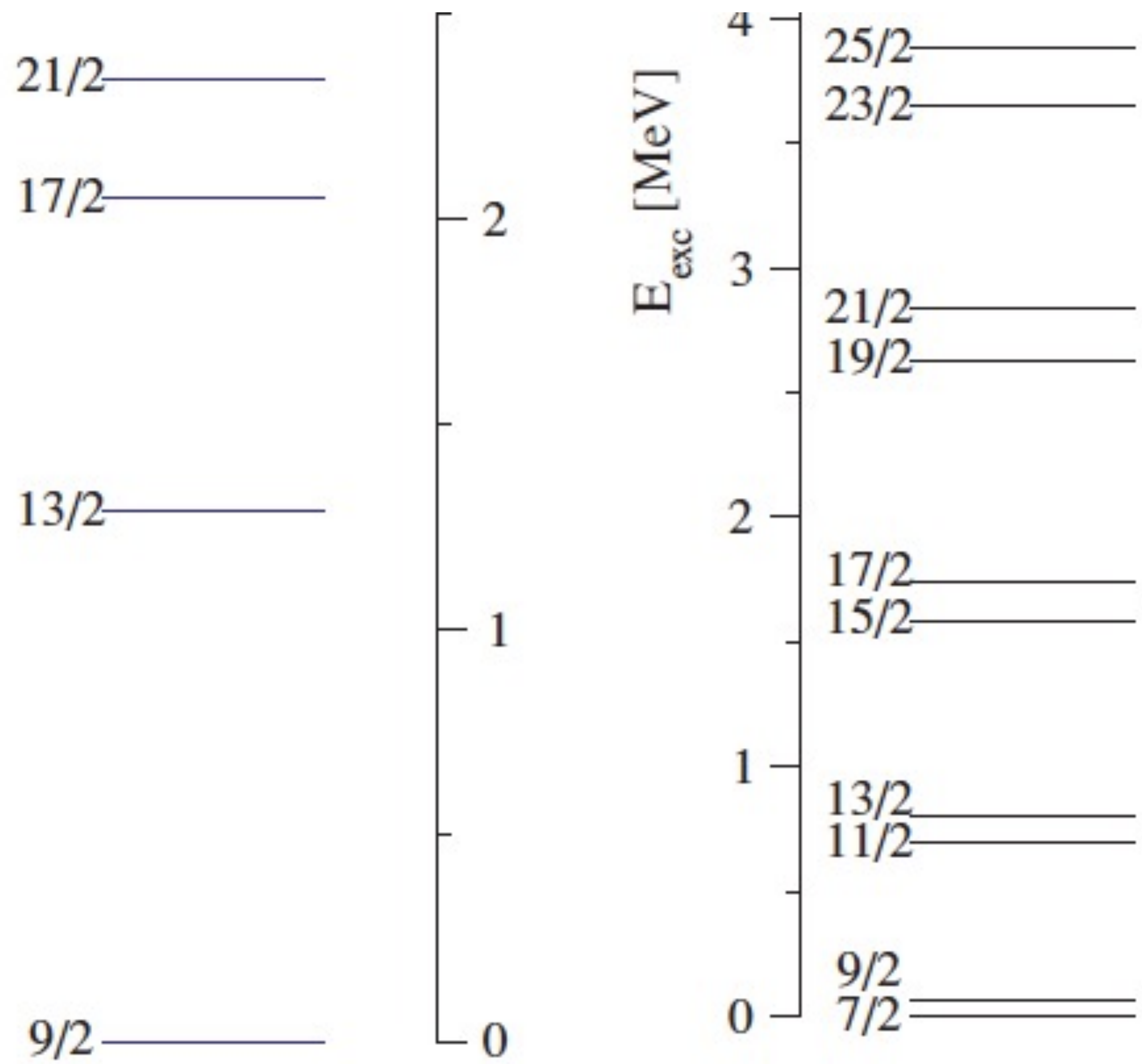
$$M = \sum_{J_A} M(J_A) E(J_A), \tag{6}$$

We finally get

$$\begin{aligned} M(J_A) \;=\; & \sum_{J_3} [j^3 J_3 j | \} j^4 I = 4 v = 2] \\ & \times [j^3 J_3 j | \} j^4 I = 4 v_a = 4] \\ & \times \left[1/3 + 2/3 \begin{Bmatrix} j & j & J_A \\ J_3 & j & J_A \end{Bmatrix} (2J_A + 1) \right] = 0 \end{aligned}$$

for $J_A = 0, 2, 4, 6$, and 8.

97AG 83ZR


21/2
17/2
13/2
9/2
2
1
0
4
3
2
1
0
E_{exc} [MeV]
25/2
23/2
21/2
19/2
17/2
15/2
13/2
11/2
9/2
7/2

Topic 3. 96Cd

- What happens in N=Z 96Cd when we set ALL T=0 2-body matrix elements to zero.

16^+ Spin-Gap Isomer in ^{96}Cd

B. S. Nara Singh,[1] Z. Liu,[2] R. Wadsworth,[1] H. Grawe,[3] T. S. Brock,[1] P. Boutachkov,[3] N. Braun,[4] A. Blazhev,[4] M. Górska,[3] S. Pietri,[3] D. Rudolph,[5] C. Domingo-Pardo,[3] S. J. Steer,[6] A. Ataç,[7] L. Bettermann,[4] L. Cáceres,[3] K. Eppinger,[8] T. Engert,[3] T. Faestermann,[8] F. Farinon,[3] F. Finke,[4] K. Geibel,[4] J. Gerl,[3] R. Gernhäuser,[8] N. Goel,[3] A. Gottardo,[2] J. Grębosz,[9] C. Hinke,[8] R. Hoischen,[3,5] G. Ilie,[4] H. Iwasaki,[4] J. Jolie,[4] A. Kaşkaş,[7] I. Kojouharov,[3] R. Krücken,[8] N. Kurz,[3] E. Merchán,[3] C. Nociforo,[3] J. Nyberg,[10] M. Pfützner,[11] A. Prochazka,[3] Zs. Podolyák,[6] P. H. Regan,[6] P. Reiter,[4] S. Rinta-Antila,[12] C. Scholl,[4] H. Schaffner,[3] P.-A. Söderström,[10] N. Warr,[4] H. Weick,[3] H.-J. Wollersheim,[3] P. J. Woods,[2] F. Nowacki,[13] and K. Sieja[13]

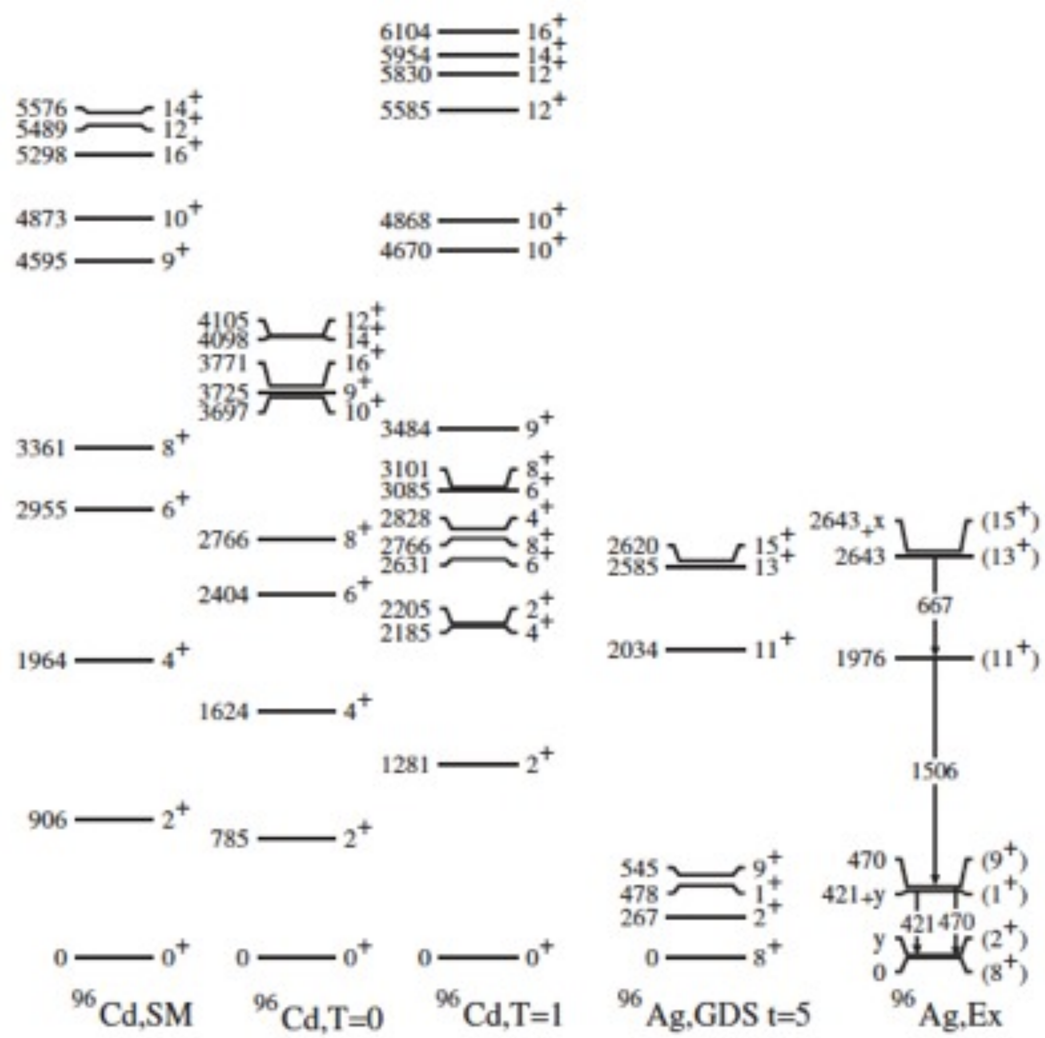


Table II: Wave functions and energies (in MeV, at the top) of selected states of Cd calculated with the INTd interaction (see text) with $T = 0$ matrix elements set to zero.

$J = 11$							
		5.0829	5.3798	6.8295	7.4699	7.5178	7.8842
J_p	J_n			$T = 1$	$T = 1$	$T = 1$	$T = 1$
4	8	0.7071	0.0000	0.2933	−0.5491	0.3351	−0.0121
6	6	0.0000	0.0000	0.2913	0.5605	0.6482	−0.4253
6	8	0.0000	0.7071	0.5350	0.0396	−0.4111	−0.2079
8	4	−0.7071	0.0000	0.2933	−0.5491	0.3351	−0.0121
8	6	0.0000	−0.7071	0.5350	0.0396	−0.4111	−0.2079
8	8	0.0000	0.0000	0.4130	0.2822	0.1319	0.8558
$J = 12$							
		5.1165	5.2336	5.4865	7.5293	7.5959	12.4531
J_p	J_n				$T = 1$	$T = 1$	$T = 2$
4	8	0.5699	0.2803	−0.0961	−0.4783	0.5208	0.2957
6	6	0.5712	−0.7151	0.1498	0.0000	0.0000	−0.3742
6	8	0.0925	0.3679	0.4629	−0.5208	−0.4783	−0.3766
8	4	0.5699	0.2803	−0.0961	0.4783	−0.5208	0.2957
8	6	0.0925	0.3679	0.4629	0.5208	0.4783	−0.3766
8	8	−0.0846	−0.2465	0.7284	0.0000	0.0000	0.6337
$J = 13$							
		5.3798	7.6143	7.8873			
J_p	J_n		$T = 1$	$T = 1$			
6	8	0.7071	0.5265	−0.4721			
8	6	−0.7071	0.5265	−0.4721			
8	8	0.0000	0.6676	0.7445			
$J = 14$							
		5.3798	5.6007	7.8515			
J_p	J_n			$T = 1$			
6	8	0.7071	0.0000	−0.7071			
8	6	0.7071	0.0000	0.7071			
8	8	0.0000	1.0000	0.0000			
$J = 15$							
		7.9251					
J_p	J_n	$T = 1$					
8	8	1.0000					
$J = 16$							
		5.6007					
J_p	J_n						
8	8	1.0000					

Table VI: Wave functions and energies (in MeV, at the top) of selected states of ^{96}Cd calculated with the interaction INTd with $T = 0$ matrix elements set to zero.

$J = 11$							
		5.0829	5.3798	6.8295	7.4699	7.5178	7.8842
J_p	J_n			$T = 1$	$T = 1$	$T = 1$	$T = 1$
4	8	0.7071	0.0000	0.2933	−0.5491	0.3351	−0.0121
6	6	0.0000	0.0000	0.2913	0.5605	0.6482	−0.4253
6	8	0.0000	0.7071	0.5350	0.0396	−0.4111	−0.2079
8	4	−0.7071	0.0000	0.2933	−0.5491	0.3351	−0.0121
8	6	0.0000	−0.7071	0.5350	0.0396	−0.4111	−0.2079
8	8	0.0000	0.0000	0.4130	0.2822	0.1319	0.8558
$J = 12$							
		5.1165	5.2336	5.4865	7.5293	7.5959	12.4531
J_p	J_n				$T = 1$	$T = 1$	$T = 2$
4	8	0.5699	0.2803	−0.0961	−0.4783	0.5208	0.2957
6	6	0.5712	−0.7151	0.1498	0.0000	0.0000	−0.3742
6	8	0.0925	0.3679	0.4629	−0.5208	−0.4783	−0.3766
8	4	0.5699	0.2803	−0.0961	0.4783	−0.5208	0.2957
8	6	0.0925	0.3679	0.4629	0.5208	0.4783	−0.3766
8	8	−0.0846	−0.2465	0.7284	0.0000	0.0000	0.6337

J=13

5.3798 7.6143 7.8873

6	8	0.7071	0.5265	-0.4721
8	6	-0.7071	0.5265	-0.4721
8	8	0.0000	0.6676	0.7445

J=14

5.3798 5.6007 7.8515

6	8	0.7071	0.0000	-0.7071
8	6	0.7071	0.0000	0.7071
8	8	0.0000	1.0000	0.0000

J=15

7.9251

8 8 1.0000

J=16

5.6007

8 8 1.0000

TALMI

j^3 configuration is given by $J_{\max} = M_{\max} = j + j - 1 + j - 2 = 3j - 3$. There is only *one* antisymmetric state with $M = M_{\max} - 1$ obtained by antisymmetrizing the state with $m_1 = j$, $m_2 = j - 1$ and $m_3 = j - 3$. Hence this is the state with $J = M_{\max}$, $M = M_{\max} - 1$ and there is no antisymmetric state with $J = 3j - 4$ in any j^3 configuration. There are two possible parents (for $j > \frac{3}{2}$) of such a state, had it existed, namely, $J_1 = 2j - 1$ and $J_1 = 2j - 3$. We can choose one of these to be the principal parent and then the c.f.p. of the other should vanish. According to (15.7) we obtain

$$\boxed{\begin{Bmatrix} j & j & 2j-1 \\ 3j-4 & j & 2j-3 \end{Bmatrix} = 0 \qquad \text{for any} \quad j > \tfrac{3}{2}}$$

If we consider states of particles in the l-orbit which are fully antisymmetric in their space coordinates we may use a similar argument. In that case, due to the symmetry properties of Clebsch-Gordan coefficients space antisymmetric states of two particles have odd values of L_0. The same considerations lead to vanishing of the $6j$-symbol written above also for values of j which are integers.

The generalization of coefficients of fractional parentage can be

Single-j-shell calculations ($f_{7/2}$)

Examples 3

- SCANDIUM 43

```
              I=6.5

              3.50013   4.95078
 JP    JN
 3.5   4.0    0.98921 -0.14647     --->    1    0
 3.5   6.0    0.14647  0.98921             0    1

              I=7.5

              3.51123   7.29248
 JP    JN                 T=3/2
 3.5   4.0    0.87905 -0.47673     --->    UNCHANGED
 3.5   6.0    0.47673  0.87905
```

- There are also degeneracy conditions
  - For ^{43}Sc, to explain the degeneracies of $13/2^-, 17/2^-$, and $19/2^-$, all with the configuration $[j, 6]$, we note

$$\begin{Bmatrix} j & j & (2j-1) \\ j & I & (2j-1) \end{Bmatrix} = \frac{-1}{8j-2}$$

    for $I = 13/2, 17/2$, and $19/2$ (but not for $I = 15/2$).
  - For ^{44}Ti, we get two degeneracy conditions:

$$\begin{Bmatrix} j & j & (2j-3) \\ j & j & (2j-1) \\ (2j-3) & (2j-1) & I \end{Bmatrix} = \frac{1}{4(4j-5)(4j-1)},$$

$$\begin{Bmatrix} j & j & (2j-1) \\ j & j & (2j-1) \\ (2j-1) & (2j-1) & I \end{Bmatrix} = \frac{1}{2(4j-1)^2}$$

    Arima and Zhao also derived these conditions using a j-pairing Hamiltonian.

Racah noted that for electrons in the f shell the calculation of coefficients of fractional parentage could be greatly simplified by noting that the exceptional group G2 is a subgroup of SO(7) [11].

The proof involved noting the following 6-j relation: $\left\{\begin{smallmatrix} 5 & 5 & 3 \\ 3 & 3 & 3 \end{smallmatrix}\right\} = 0$. Regge [12] found several symmetry relations for 6-j symbols, one of which is

$$\left\{\begin{matrix} a & b & e \\ d & c & f \end{matrix}\right\} = \left\{\begin{matrix} a & 1/2(b+c+e-f) & 1/2(b-c+e+f) \\ d & 1/2(b+c-e+f) & 1/2(-b+c+e+f) \end{matrix}\right\}. \tag{20}$$

Early on, Judd and Elliott [13] used this to show that

$$\left\{\begin{matrix} 5 & 5 & 3 \\ 3 & 3 & 3 \end{matrix}\right\} = \left\{\begin{matrix} 5 & 4 & 4 \\ 3 & 4 & 2 \end{matrix}\right\}. \tag{21}$$

See also the work of Judd and Li [1]. Furthermore we emphasized at the beginning of this work that for quartet states of three electrons in the g shell the space wave function has to be antisymmetric. This leads to the vanishing of the 6-j on the right-hand side above. This is easier to understand than

- Re 44Sc and 52Mn (3proton holes and one neutron hole)
- But we find that in the light member of the cross-conjugate pair the J=2+ state . In 52Mn the J=6+ state is ground state .
- But in both nuclei J=2+ and J=6+ are isomeric.
- We find (2j-1)rule--State with (2j-1) is isomeric.

- (2j-1)rule and J=2+ rule.
- j=7/2: J=6+ state in 52Mn is ground state
- and J=2+ is isomeric.
- Also in 44Sc J=6+ is isomeric and J=2+ is ground state.
- In 96Ag (g9/2) (2j-1)=8+ and J=2+ are nearly degenerate and both are isomeric
- In h11/2 shell J=2+ and J=10+ isomeric

Table VII: Two-body matrix elements in increasing spin from $J = 0$ to $J = J_{\max}$. The even spins have isospin $T = 1$ and the odd ones $T = 0$.

	$f_{7/2}$		$g_{9/2}$		$h_{11/2}$
J	INTa	INTb	INTc	INTd	$Q \cdot Q$
0	0.0000	0.0000	0.0000	0.0000	−1.0000
1	0.6111	0.5723	1.1387	1.1387	−0.9161
2	1.5863	1.4465	1.3947	1.3947	−0.7544
3	1.4904	1.8224	1.8230	1.8230	−0.5325
4	2.8153	2.6450	2.0823	2.0823	−0.2687
5	1.5101	2.1490	1.9215	1.9215	0.0070
6	3.2420	2.9600	2.2802	2.2802	0.2587
7	0.6163	0.1990	1.8797	1.8797	0.4434
8			2.4275	2.4275	0.5105
9			1.4964	0.7500	0.4026
10					0.0549
11					−1.6044

Table I: Yrast spectra of ^{44}Ti and ^{52}Fe calculated with the interactions INTa and INTb respectively (see text) and compared with experiment [8].

	E(MeV)			
	^{44}Ti		^{52}Fe	
J	INTa	Exp.	INTb	Exp.
0	0.000	0.000	0.000	0.000
1	5.669		5.442	
2	1.163	1.083	1.015	0.849
3	5.786		5.834	
4	2.790	2.454	2.628	2.384
5	5.871		6.463	
6	4.062	4.015	4.078	4.325
7	6.043		5.890	
8	6.084	(6.509)	5.772	6.361
9	7.984		7.791	
10	7.384	(7.671)	6.721	7.382
11	9.865		8.666	
12	7.702	(8.040)	6.514	6.958

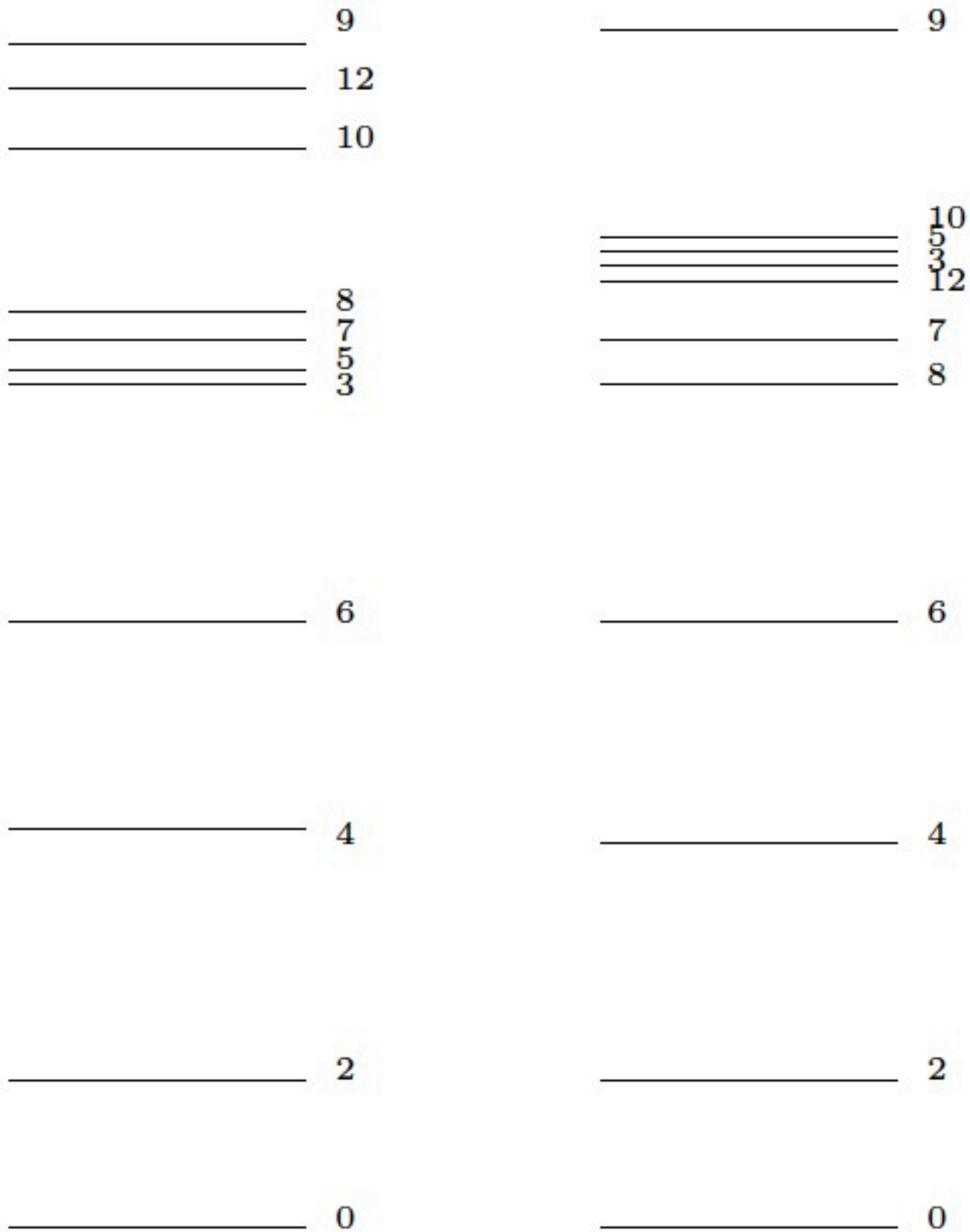

9
12
10
8
7
5
3
6
4
2
0
9
10
5
3
12
7
8
6
4
2
0

Table II: Yrast spectra of ^{44}Sc and ^{52}Mn calculated with the interactions INTa and INTb respectively (see text) and compared with experiment [8].

	E(MeV)			
	^{44}Sc		^{52}Mn	
J	INTa	Exp.	INTb	Exp.
0	3.047		2.774	
1	0.432	0.667	0.443	0.546
2	0.000	0.000	0.202	0.378
3	0.764	0.762	0.836	0.825
4	0.713	0.350	0.851	0.732
5	1.276	1.513	1.404	1.254
6	0.381	0.271	0.000	0.000
7	1.272	0.968	1.819	0.870
8	3.097		2.572	(2.286)
9	3.390	2.672	2.792	(2.908)
10	4.793	4.114	4.365	4.164
11	4.638	3.567	3.667	(3.837)

Table III: Energy levels for the case of 3 protons and 1 neutron in the $g_{9/2}$ shell with the interactions INTc and INTd (see text), and compared with the experimental data for ^{96}Ag.

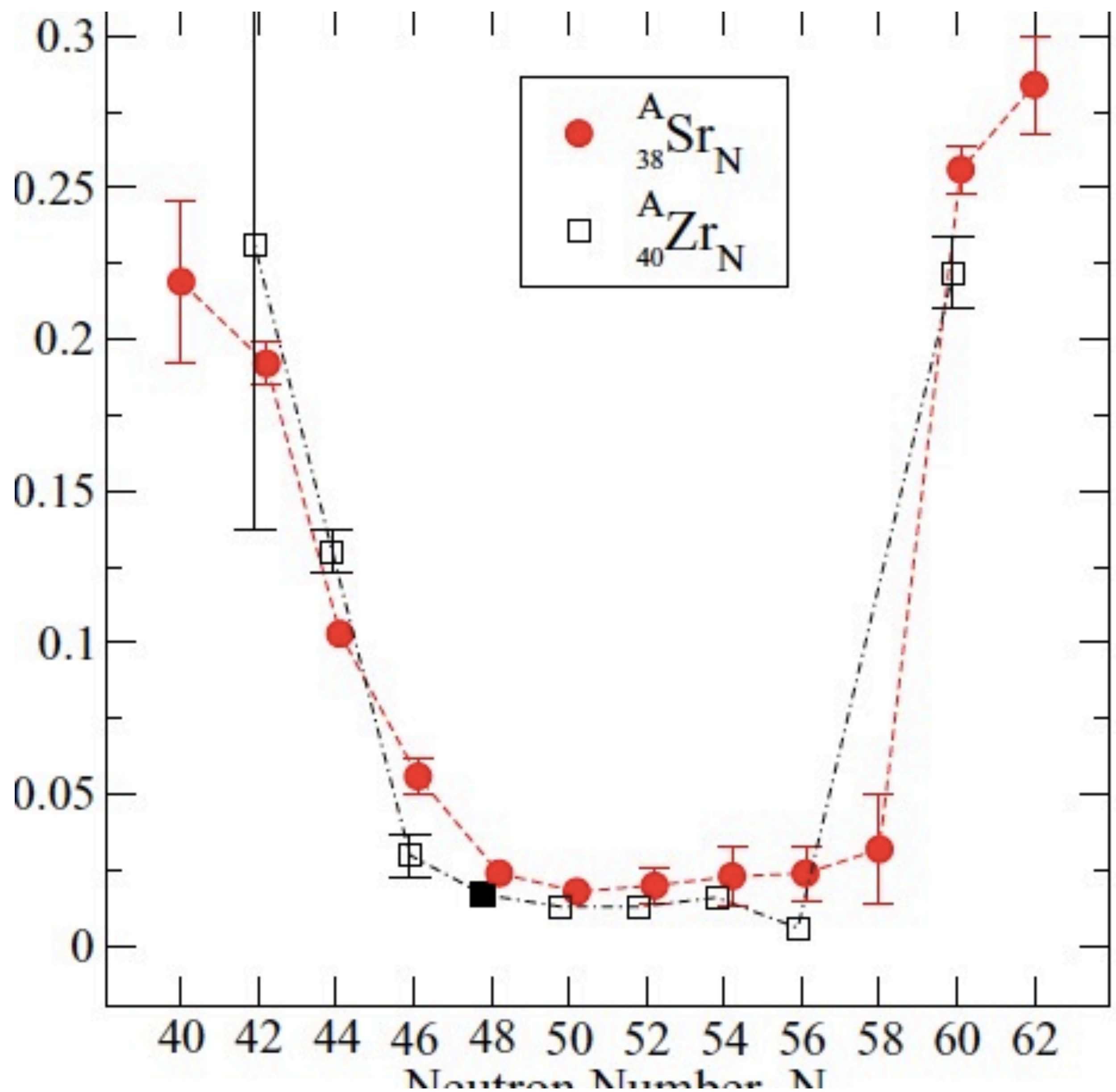
	E(MeV)		
J	INTc	INTd	Exp.
0	0.246	0.900	
1	0.463	0.483	
2	0.000	0.097	
3	0.638	0.588	
4	0.394	0.349	
5	0.774	0.737	
6	0.450	0.371	
7	0.850	0.861	
8	0.350	0.000	0.000
9	0.872	0.492	0.470
10	2.188	1.748	(1.719)
11	2.344	1.930	(1.976)
12	3.004	2.550	
13	3.087	2.556	2.643
14	3.382	3.070	
15	3.287	2.645	$2.643+x$

- STATIC MAGNETIC MOMENTS (Rutgers-Bonn) AND QUADRUPOLE MOMENTS OF EXCITED STATES

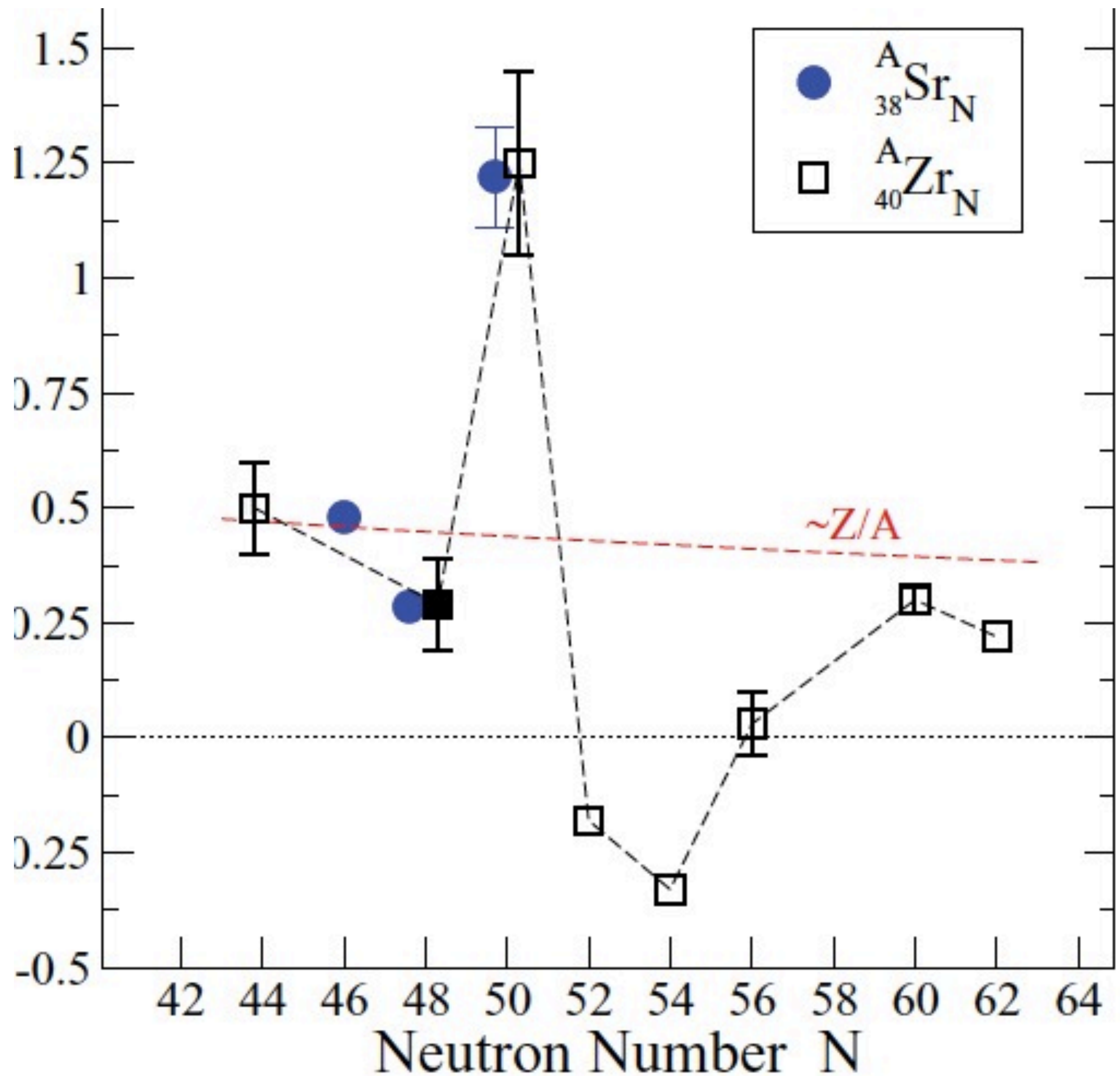
TABLE V. Calculated and experimental $g(I)$ factors in ^{70}Zn.

I_i^π	Exp't.	FPD6 fp	KB3 fp	GXPFA fp	JJ4B $p_{3/2}f_{5/2}p_{1/2}g_{9/2}$
2_1^+	+0.38(2)[a]	+1.52	+1.83	+1.89	+0.276
2_2^+	+0.47(22)	+1.26	+1.99	+1.53	+0.100
4_1^+	+0.37(14)	+1.12	+1.18	+1.12	+0.317

[a] A value, $g = +0.38(4)$, was obtained in Ref. [1].



0.3
0.25
0.2
0.15
0.1
0.05
0
$^{A}_{38}Sr_N$
$^{A}_{40}Zr_N$
40 42 44 46 48 50 52 54 56 58 60 62



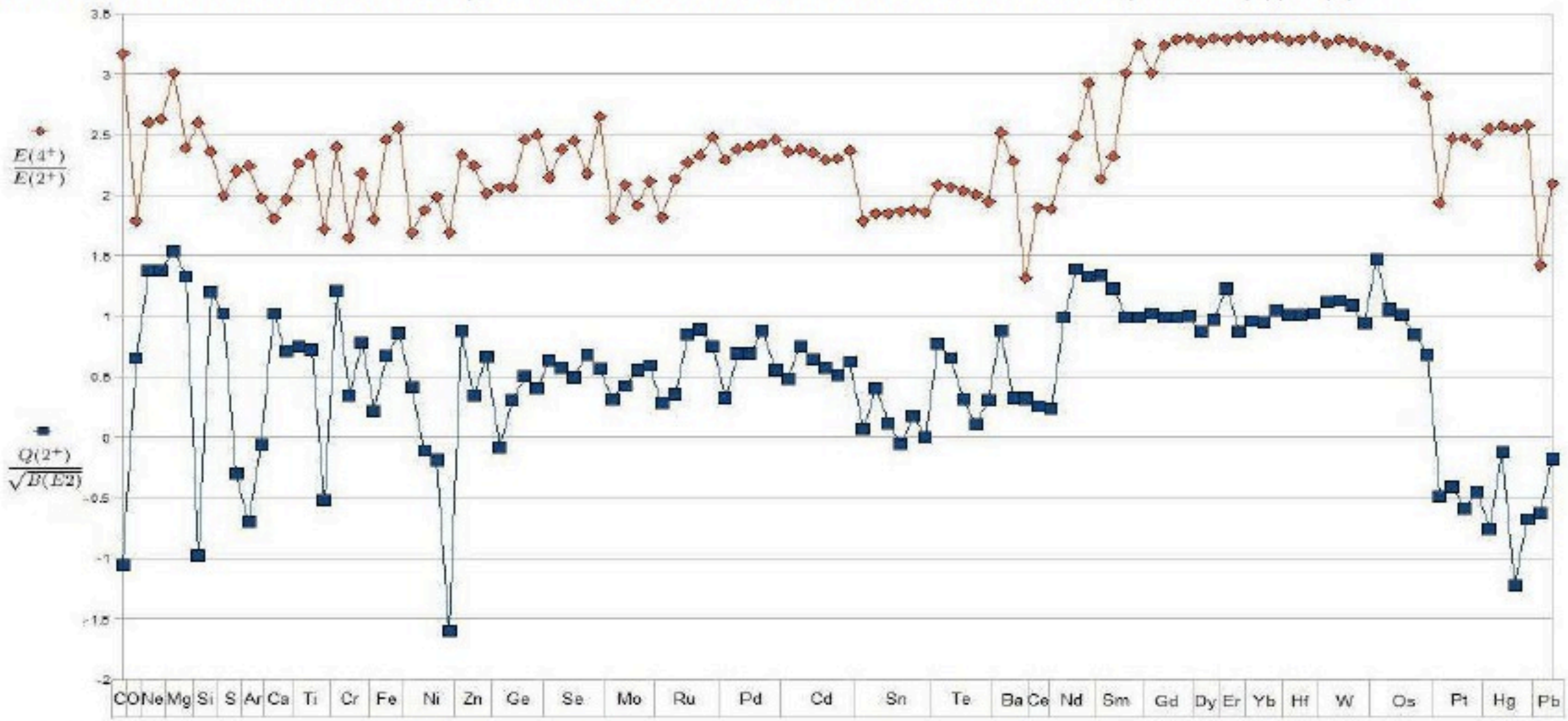
1.5
1.25
1
0.75
0.5
0.25
0
-0.25
-0.5
${}^{A}_{38}Sr_N$
${}^{A}_{40}Zr_N$
~Z/A
42 44 46 48 50 52 54 56 58 60 62 64
Neutron Number N

TABLE III. Static quadrupole moments in e fm^2. Two sets of effective charges are used $e_p = 1.5$ and $e_n = 0.5$(displayed first) and $e_p = 1.5$ and $e_n = 1.1$. N/A indicates unavailable data; references are given for available experimental data.

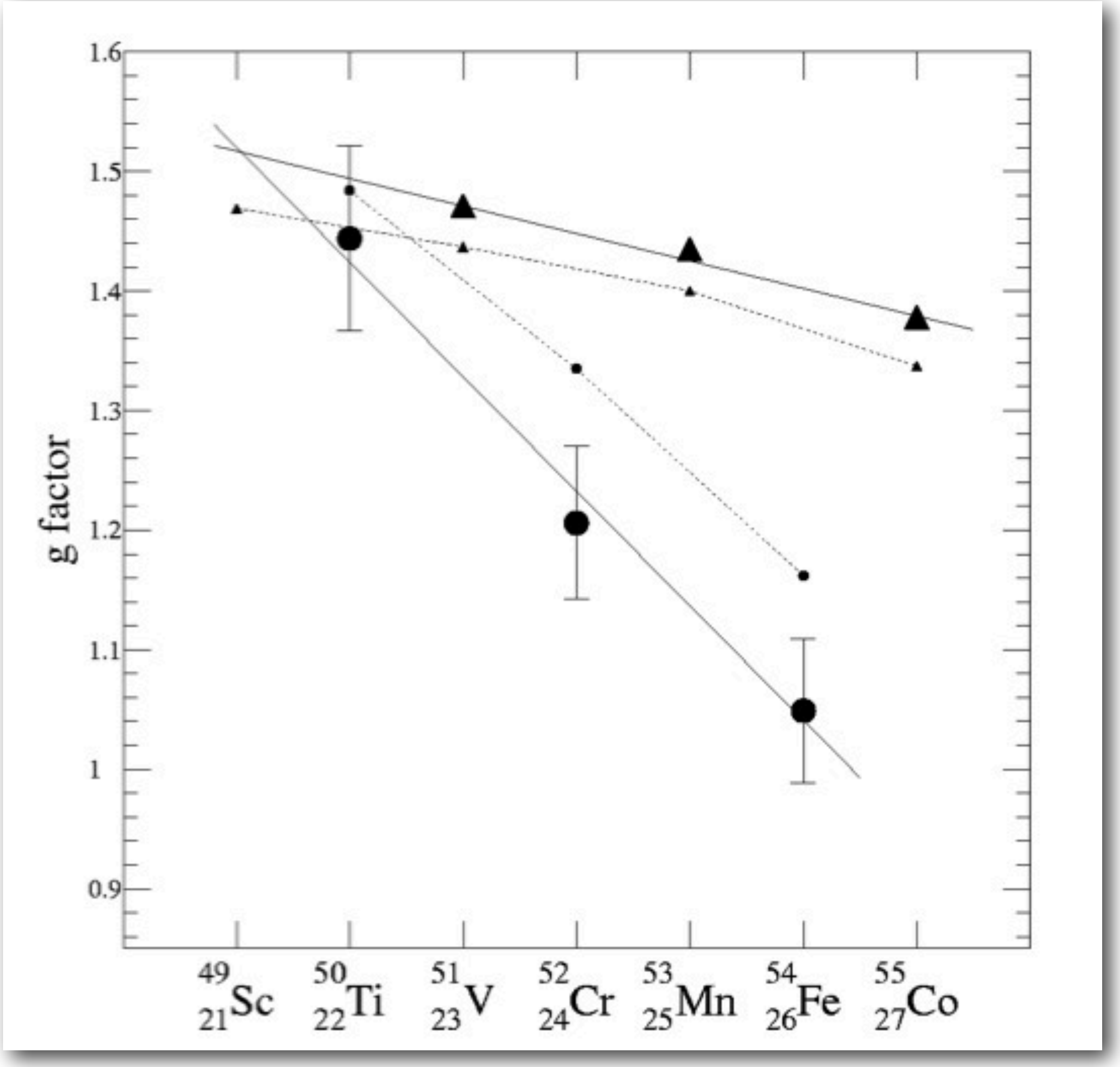
	^{70}Ge	^{72}Ge	^{74}Ge	^{76}Ge
$Q(2_1^+)$				
Expt.	+4(3) [13]	−12(8) [14]	−19(2) [15]	−14(4) [16]
	+3(6) or 9(6) [17]	−13(6) [17]	−25(6) [17]	−19(6) [17]
JJ4B	+15/+25	+11/+19	−6/−6	−15/−19
JUN45	+10/+17	+13/+22	+12/+20	+2/+5
$Q(2_2^+)$				
Expt.	−7(4) [13]	+23(8) [14]	+26(6) [15]	+28(6) [16]
JJ4B	−15/−25	−11/−19	+5/+6	+15/+20
JUN45	−13/−21	−13/−22	−12/−19	−0.1/−2
$Q(4_1^+)$				
Expt.	+22(5) [13]	N/A	N/A	−1 (5) [16]
JJ4B	+3/+11	+3/+10	−8/−9	−14/−17
JUN45	+1/+8	+8/+8	+11/+19	−1/+1

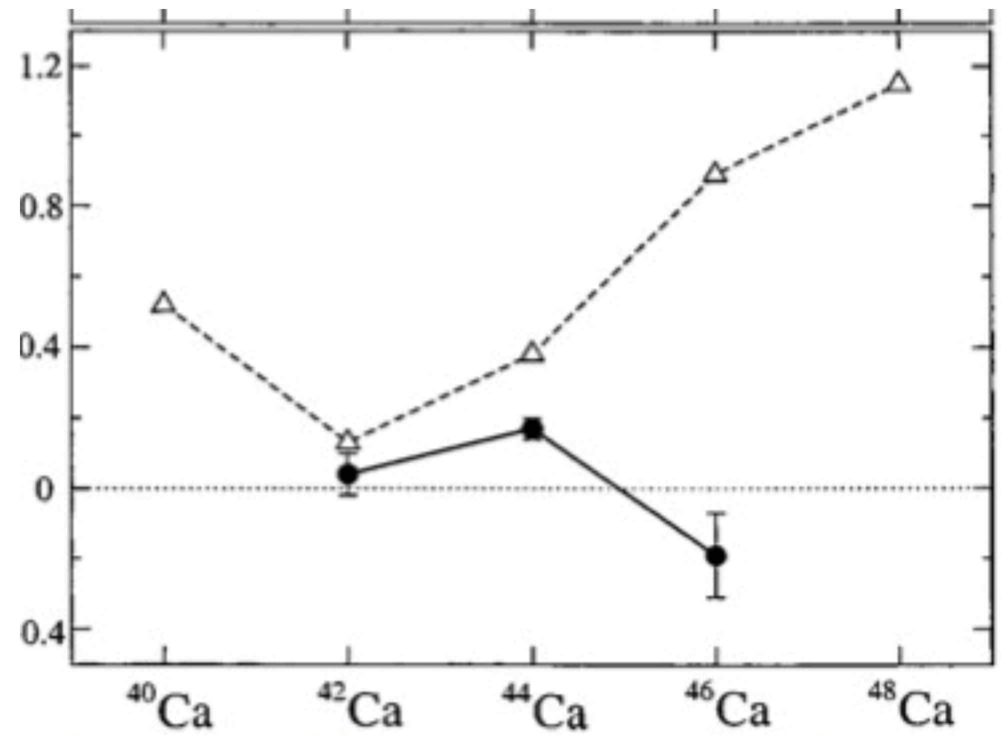
III. RESULTS

In Table I we present for even-even nuclei the magnitude ofthe quadrupole ratio and $E(4)/E(2)$, the ratio of energies. Again, in the pure rotation model r_Q is equal to 1 and $E(4)/E(2) = \dfrac{4 \times 5}{2 \times 3} = 3.33$. In a simple vibrational limit the quadrupole moment would be zero (i.e. the static quadrupole moment would vanish) and $E(4)/E(2) = 2$.



The results are also shown in Fig 1. The upper curve is the ratio $E(4)/E(2)$ and the lower curve is the ratio involving $Q(2^+)$ and $B(E2)$.


g factor
1.6
1.5
1.4
1.3
1.2
1.1
1
0.9
$^{49}_{21}$Sc
$^{50}_{22}$Ti
$^{51}_{23}$V
$^{52}_{24}$Cr
$^{53}_{25}$Mn
$^{54}_{26}$Fe
$^{55}_{27}$Co



3. Summary of 2_1^+ excitation energies, $B(E2)$'s (in Weis-
its) and $g(2_1^+)$ factors for all stable even-A Ca isotopes (see
[4]). Closed circles refer to our present and former data
circles to Ref. [3,5]. The g factors are compared to the

ANATOMY of an INVERSION

- As we go through the even even Argon isotopes 2 different interactions nicely track for A=36,38,40,42 and 44 but suddenly diverge for A=46

TABLE III. g factors in the even-even argon isotopes.

g factors	^{38}Ar	^{40}Ar	^{42}Ar	^{44}Ar	^{46}Ar
$g(2_1^+)$					
Experiment	0.24(12)	−0.02(2)			
Small space WBT	0.083	−0.441	−0.455	−0.441	0.083
$0\hbar\,\omega$ WBT	0.308	−0.197	−0.095	−0.022	+0.100
$0\hbar\,\omega$ SDPF-U	0.319	−0.228	−0.084	−0.040	0.513
$g(2_2^+)$					
Experiment					
Small space WBT	N/A	−0.046	−0.481	−0.046	N/A
$0\hbar\,\omega$ WBT	1.198	0.120	0.096	0.045	−0.070
$0\hbar\,\omega$ SDPF-U	1.187	0.136	0.075	0.346	−0.514
$g(4_1^+)$					
Experiment					
Small space WBT	N/A	−0.490	−0.509	−0.490	N/A
$0\hbar\,\omega$ WBT	1.134	−0.354	−0.277	−0.206	−0.190
$0\hbar\,\omega$ SDPF-U	1.132	−0.357	−0.289	−0.246	−0.388

TABLE VI. The $J = \frac{3}{2}^+$-$J = \frac{1}{2}^+$ splittings of the odd K isotopes in MeV.

	Experimental	WBT	SDPF-U
^{41}K	0.980476	1.106	0.854
^{43}K	0.5612	1.109	0.672
^{45}K	0.4745	0.871	0.345
^{47}K	−0.3600	0.507	−0.320
^{49}K	0.200	0.729	0.078

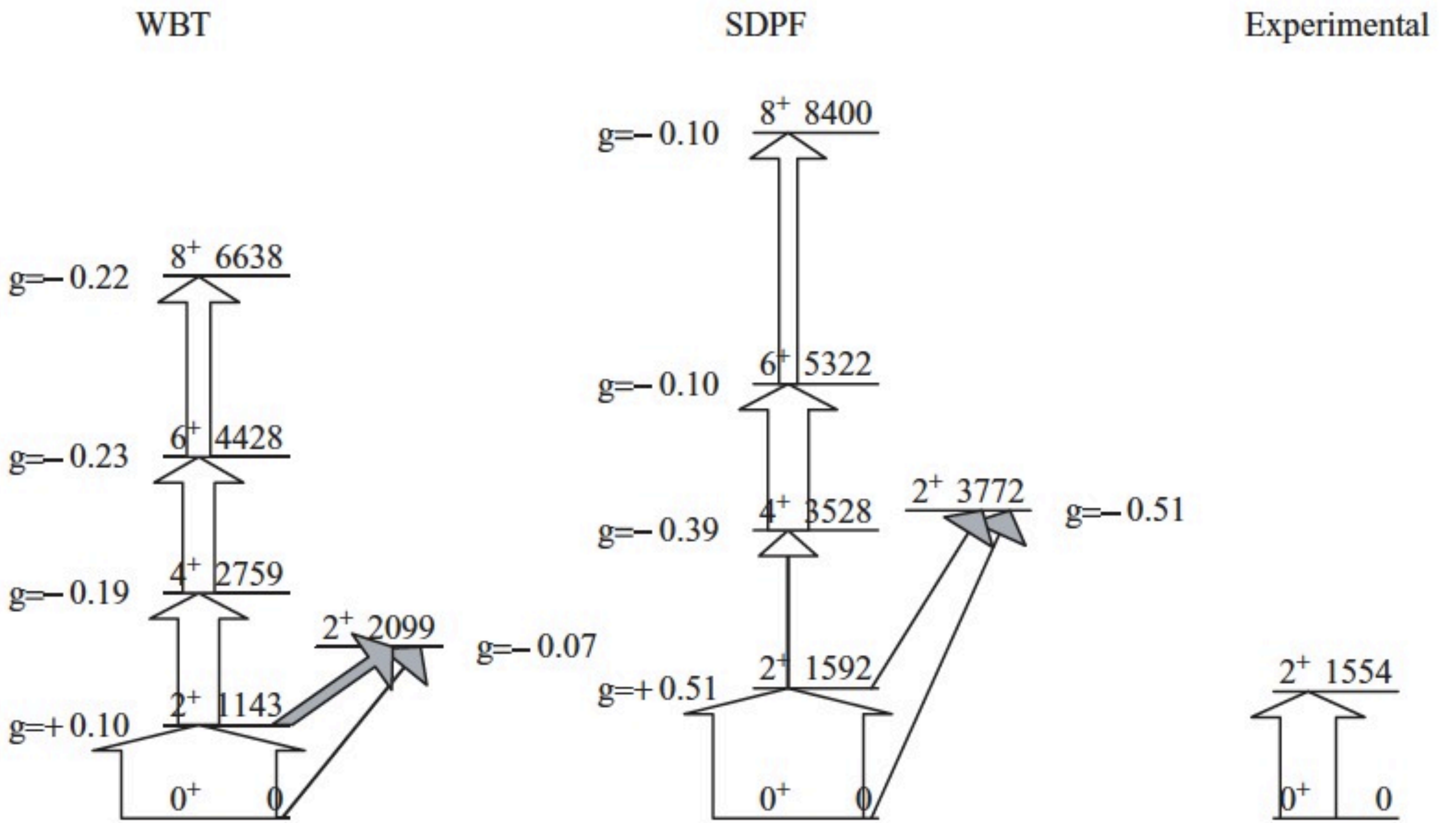


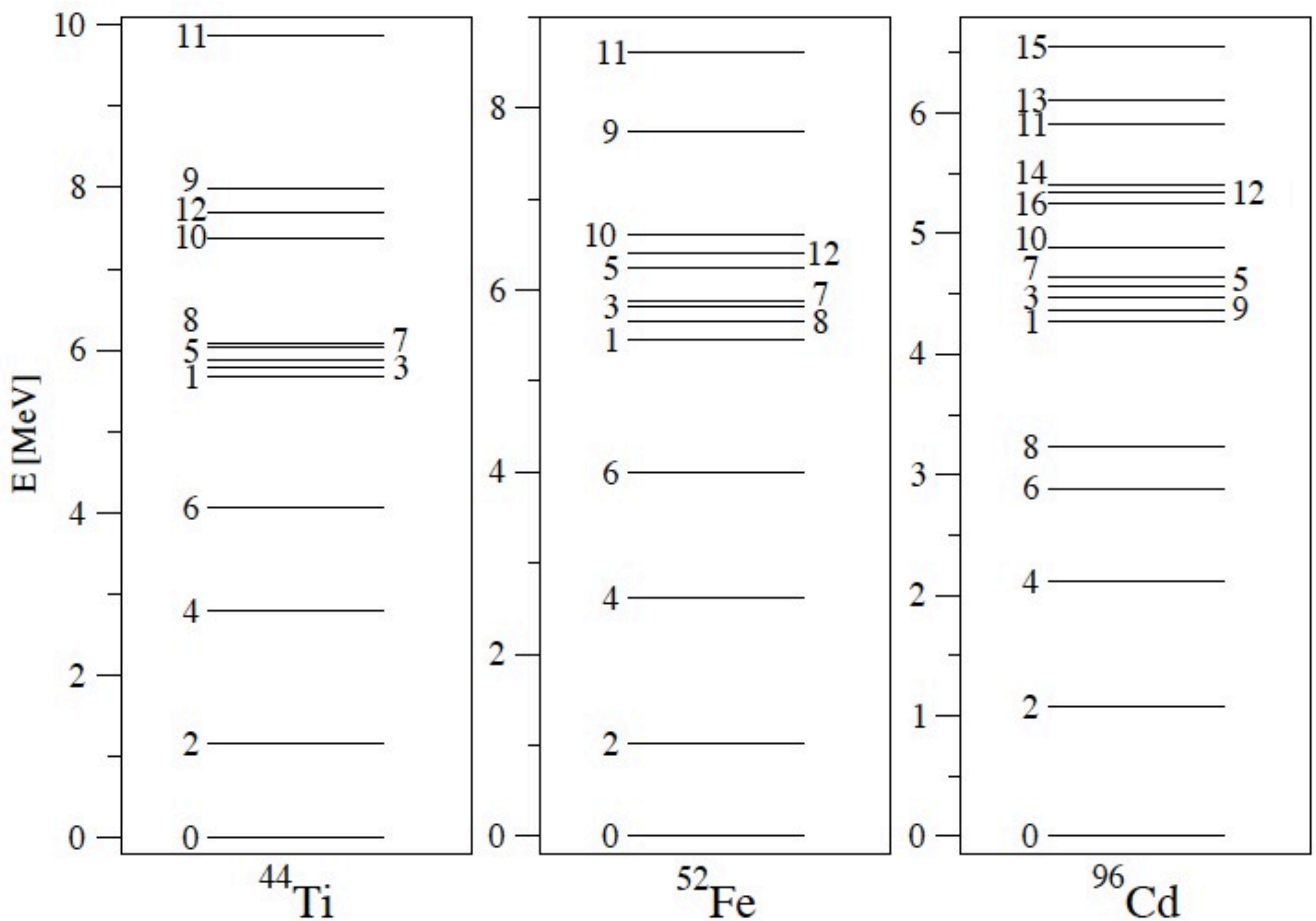
FIG. 5.
excitation e
$B(E2;\Uparrow)$ is
arrow.

$$E^*(J=0^+,T=2) = BE(^{96}\mathrm{Ag}) - BE(^{96}\mathrm{Pd}) + V_C\,, \qquad (1)$$

where V_C includes all charge-independant violating effects. We here assume that V_C arises from the Coulomb interaction and use the formula of Anderson et al. [1]:

$$V_C = E_1 Z/A^{(1/3)} + E_2\,, \qquad (2)$$

where $Z = (Z_1 + Z_2)/2$.


E [MeV]
44Ti
52Fe
96Cd
11
9
12
10
8
7
5
3
1
6
4
2
0
15
13
14
16

- Partial dynamical symmetries
- unique (g9/2)^4 state v=4 J=4 (6)
- Symmetries for J values in 96Cd which are not present in 96Pdwhen T=0 two body matrix elements set equal to zero.
- (2j-1) rule 3 protons -1neutron
- Static moments in “g9/2” shell.